LOYOLA COLLEGE (AUTONOMOUS) CHENNAI 600 034

M. Sc. Degree Examination – MATHEMATICS

First Semester – November 2014

MT 1818 – DIFFERENTIAL GEOMETRY

Date: Max: 100 Marks

Time: Dept. No.

Answer ALL the Questions:

1. a) Prove that the curvature is the rate of change of angle of contingency with respect to the arc length. (5)

OR

b) Consider the curve $\vec{r} = (u, u^2, u^3)$. Find $\vec{t}, \vec{n}, \vec{b}$ at the point $u = 1$. Also find the equations of the tangent, principal normal and binormal at $u = 1$. (5)

c) Find the equation of the osculating plane at a point of the curve of intersection of the surfaces $f(x, y, z) = 0$ and $g(x, y, z) = 0$. (15)

OR

d) Derive the Serret-Frenet formulae. Express them in terms of Darboux vector. (15)

2. a) Prove that the necessary and sufficient condition for a curve to be helix is that the ratio of curvature to torsion is constant. (5)

OR

b) Find the lines that have four point contact at (0, 0, 1) with the surface $x^4 + 3xyz + x^2 - y^2 - z^2 + 2yz - 3xy - 2y + 2z = 1$. (5)

c) Derive the equation of involute. Also find the equation of curvature and torsion of an involute. (15)

OR

d) Find the curve whose intrinsic equations are $\kappa = \frac{1}{\sqrt{2as}}$ and $\tau = 0$. (15)

3. a) Prove that the first fundamental form is a positive definite. (5)

OR

b) Prove that the metric is invariant under a parametric transformation. (5)

c) Prove that the necessary and sufficient condition for the surface may be developable is that its Gaussian curvature surface is zero. (15)

OR

d) Define and derive polar and rectifying developable associated with a space curve. (15)

4. a) Derive the equation satisfying principal curvature at a point on a surface. (5)

OR

b) If $h(t)$ is a continuous function for $0 < t < 1$ and if $\int_0^1 v(t)h(t)dt = 0$, for all admissible function $v(t)$, then prove that $h(t) = 0$. (5)

c) Prove that on a general surface, a necessary and sufficient condition for the curve $v = c$ to be a geodesic is that $EE_2 + FE_1 - 2EF_1 = 0$for all values of the parameter. (15)

OR

d) State and prove Euler's theorem. (15)

5. a) State and prove Liebmann theorem. (5)

OR

b) Derive Rodrigue's formula for the lines of curvature from Weingarten equations. (5)

c) Derive Mainardi-Codazzi equations. (15)

OR

d) State and prove the fundamental theorem of surface theory and demonstrate. (15)

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